Math 100 HW 1

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Section 1.1

Q10) We are asked to find {x $\in \mathbb{R}$: $cos(x) = 1\}$ Note that cos(x)=1 iff $x = 2k\pi$ Thus are list will be $\{\ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi \ldots\}$

Q16) We are asked to find {6a + 2b: $a, b \in \mathbb{Z}\}$ There are a couple of ways of going about this: One way is to first simplify the expression. Note 6a+2b= 2(3a +b). So right away we can see this list will only consist of even integers. The question is whether or not EVERY even number is in this list. We show it is: let n=2k be any even integer. Then n=6(0) + 2k so we see that n is in fact in this list (with a=0, b=k). So this set is really just every even integer. Thus the answer is $\{\ \ldots, -4, -2, 0, 2, 4, \ldots\}$

Q26) We are given the set $\{\ \ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27 \ldots\}$ Note that all these are powers of 3. Therefore we get that this set is equal to the set $\{3^n : n \in \mathbb{Z}\}$

Section 1.2

Q2(g)) we are given that A= $\{\pi, e, 0\}$ and B= {0,1} We are asked to find $A \times (B \times B)$. We will follow the order of operations and first do $B \times B$ (although, ask yourself if you think it matters which we do first). We get that $B \times B$= {(0,0), (0,1), (1,0), (1,1)} Thus

$$\begin{aligned}\mathcal{A} \times (\mathcal{B} \times \mathcal{B}) = \{\ &(\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1), (e, 0, 0), (e, 0, 1),\\ &(e, 1, 0), (e, 1, 1), (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)\}\end{aligned}$$

Q8) We are asked to find $\{0, 1\}^4$. Recall this means finding {0,1} $\times\{0, 1\} \times \{0, 1\} \times \{0, 1\}$ For convenience, let A= {0,1}. Then we will find A $\times A$ twice:

indeed A $\times A$= {(0,0), (0,1), (1,0), (1,1)} Thus

$$\begin{aligned}\mathcal{A}\times\mathcal{A}\times\mathcal{A}\times\mathcal{A} = \{ & (0,0,0,0),(0,0,1,0),(0,0,0,1),(0,0,1,1),\\ & (0,1,1,0),(0,1,1,1),(0,1,0,0),(1,0,1,1),\\ & (1,0,1,0),(1,0,0,1),(1,0,0,0),(1,1,1,0),\\ & (1,1,0,1),(1,1,0,0),(1,1,1,1),(0,1,0,1)\}\end{aligned}$$

Section 1.3

Q6) We want to find all the subsets of the set $\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$ Each individual element considered as a set will be a subset: so we get the three subsets $\{\mathbb{R}\},\{\mathbb{Q}\},\{\mathbb{N}\}$. Now we can take combinations of the elements to get the remaining subsets $\{\mathbb{R},\mathbb{Q}\},\{\mathbb{R},\mathbb{N}\},\{\mathbb{Q},\mathbb{N}\},\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$ and finally the empty set $\emptyset$. This gives us 8 subsets in total, or 2^3 subsets as expected.

Q16) We are asked to determine if the set $\{(x,y) : x^2 - x = 0\} \subseteq \{(x,y) : x - 1 = 0\}$. Call the first set A, and the second set B for convienence. Then recall A $\subseteq B$ iff every element of A is also an element of B. Yet note the element (0,0) $\in$ A yet (0,0) $\notin B$ so we conclude that A $\not\subseteq$ B. As an aside, is B $\subseteq$ A?

Section 1.4

Q6) We want to find the set $\mathcal{P}(\{1,2\})\times\mathcal{P}(\{3\})$. We will do this step by step: first we find that A=$\mathcal{P}(\{1,2\})$ = {{1}, {2}, {1,2}, $\emptyset$}; second we find that B=$\mathcal{P}(\{3\})$= {{3}, $\emptyset$} Taking their cartesian products we get

$$\begin{aligned}A\times B = \{ & (\{1\},\{3\}),(\{2\},\{3\}),(\{1,2),\{3\}),\\ & (\emptyset,\{3\}),(\{1\},\emptyset),(\{2\},\emptyset),(\{1,2\},\emptyset),(\emptyset,\emptyset)\}\end{aligned}$$

Q18) We are asked to find $|\mathcal{P}(A\times\mathcal{P}(B))|$ and are told that $|A| = m$ and $|B| = n$. The two relevant facts here are:

$$\text{given a set C, } |\mathcal{P}(C)| = 2^{|C|} \tag{1}$$

and that

$$\text{given sets C and D, } |C\times D| = |C|\cdot|D| \tag{2}$$

So if we can find the cardinality of $A\times\mathcal{P}(B)$, we can find the entire cardinality by doing 2 to that value. Indeed from (1) we have that $|\mathcal{P}(B)| = 2^n$

and from (2) we get that $|A \times \mathcal{P}(B)| = m \cdot 2^n$ Finally, using (1) again, we get that $|\mathcal{P}(A \times \mathcal{P}(B))| = 2^{m \cdot 2^n} = 2^m \cdot 2^{2^n}$

Section 1.5

Q2(e)) We are told that B= {1,3,5,7} and A= {0,2,4,6,8} and asked to find B $\setminus$ A. This means we want all the elements of B that are not in A. However, notice that A $\cap$ B= $\emptyset$ so there every element of B is not in A, and hence we get that B $\setminus$ A= B.

Q2(h)) We are told that C= {2,8,4}, and A is as above, and are asked to find C $\setminus$ A. Note that every element of C is also in A, so the set of all elements in C that are not in A is empty, in other words C $\setminus$ A= $\emptyset$