Homework 2 Answers
David Rubinstein - Math 100 - Fall 2019

1.8 - 10a) Find $\bigcup_{x\in[0,1]} [x,1]\times[0,x^2]$

Solution: Each element of this union is of the form $[x,1]\times[0,x^2]$ for some x $\in [0,1]$, ie some 2-dimensional plane inside $\mathbb{R}^2$. Then finding the union of these Cartesian products is just finding the union of a bunch of nested planes. Therefore we just need to find the biggest x-interval and biggest y-interval to find the union. (I recomend drawing what these Cartesian products look like for a couple different values of x to see what I mean). We will plug in the extreme points to see what we get.
For $x = 0 : [x,1]\times[0,x^2] = [0,1]\times[0,0] = [0,1]\times\{0\}$
For $x = 1 : [x,1]\times[0,x^2] = [1,1]\times[0,1] = \{1\}\times[0,1]$
Therefore just taking the union of these two values of x we get $[0,1]\times[0,1]$. Yet notice that $[0,1]$ is the biggest possible interval size for both $[x,1]$, & $[0,x^2]$ (for $x \in [0,1]$). Thus $\bigcup_{x\in[0,1]} [x,1]\times[0,x^2] = [0,1]\times[0,1]$

(1.8- 10b) Find $\bigcap_{x\in[0,1]} [x,1]\times[0,x^2]$

Solution: This is the exact same argument as in part a; only this time we aren't looking for the largest plane, but the smallest plane; ie, the plane that each of the planes have as a subset. Again by viewing the extreme points and making a similar argument as in part a we get that $\bigcap_{x\in[0,1]} [x,1]\times[0,x^2] = \{1\}\times\{0\}$
The best way to convince yourself this is correct is by drawing out a few of the intervals and see what points they have in common.

(2.1- 8) Determine if the following is a statement. If it is a statement say if it's true or false.
$\mathbb{N} \in \mathcal{P}(\mathbb{N})$

Solution: This is a statement: it is either true or false that $\mathbb{N}$ is an element of its power set. Recall that $X \in \mathcal{P}(A)$ iff X $\subseteq A$. Then since $\mathbb{N} \subseteq \mathbb{N}$ we get that it is true that $\mathbb{N} \in \mathcal{P}(\mathbb{N})$

(2.1- 14) Call me Ishmael.

Solution: This is not a statement. If it was phrased as "My name is Ishmael" then it would be a statement.

(2.2- 8) Express the following sentence in one of the forms $P \vee Q, P \wedge Q, \neg P$
At least one of the numbers x and y equals 0.

Solution: Let P= x equals 0, and Q= y equals 0. Then the statement At least one of the numbers x and y equals 0= $P \vee Q$.

(2.3- 2) Without changing their meanings, convert each of the following sentences into a sentence having the form "If P, then Q."
For a function to be continuous, it is sufficient that it is differentiable.

Solution: Let P be the statement: a function is differentiable and Q be the statement: a function is continuous. Then the statement above says For Q it is sufficient P; in other words if P then Q (ie; if a function is differentiable, then it is continuous).

(2.3- 10) "The discriminant is negative only if the quadratic equation has no real solutions."

Solution: This statement is equivalent to the following: If the discriminant is negative then the quadratic equation has no real solutions. This is a case of the general fact that P only if Q is the same as if P then Q. Almost all of you got this one wrong. Come talk to me if you are still confused by it, this is a tricky one forsure, but one where we really need to understand what's going on.

(2.4- 2) Without changing their meanings, convert each of the following sentences into a sentence having the form "P if and only if Q."
If a function has a constant derivative then it is linear, and conversely.

Solution: This is equivalent to the statement: a function has constant derivative if and only if it is linear.

(2.5-6) Write a truth table for $(P \wedge \neg P) \wedge Q$

Solution:

P	Q	$P \wedge \neg P$	$(P \wedge \neg P) \wedge Q$
T	F	F	F
T	T	F	F
F	T	F	F
F	F	F	F

We see that this is always false since $P \wedge \neg P$ is always false.

(2.5- 8) Write the truth table for $P \vee (Q \wedge \neg R)$

Solution:

P	Q	R	$Q \wedge \neg R$	$P \vee (Q \wedge \neg R)$
T	T	T	F	T
T	F	T	F	T
T	T	F	T	T
T	F	F	F	T
F	T	T	F	F
F	T	F	T	T
F	F	T	F	F
F	F	F	F	F

(2.5- 10) Suppose the statement $((P \wedge Q) \vee R) \implies (R \vee S)$ is false. Find the values of P,Q,R,S.

Solution: This is probably the hardest problem so far in my opinion: let's take it step by step.
Notice that if $R \vee S$ were true then the whole statement would be true: so we can conclude that both R and S are false.
Now if the LHS of the implies arrow was false, the statement would be true, so we can conclude that $((P \wedge Q) \vee R)$ is true. Since we Know R is false, we can conclude that P and Q are both True.
We summarize this as follows
P= True
Q=True
R=False
S=False

(2.6- 4) Use truth table to show the following are logically equivalent.
$\neg(P \vee Q) = (\neg P) \wedge (\neg Q)$

Solution:

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P) \wedge (\neg Q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Thus we see they are logically equivalent.

(2.6- 12) Decide whether $\neg(P \implies Q) = P \wedge \neg Q$

Solution: We show these are logically equivalent by making a truth table.

P	Q	$P \implies Q$	$\neg(P \implies Q)$	$P \wedge \neg Q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

(2.7- 8) Write the following in English. Say whether they are true or false.
$\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}$ such that$|X| = n$.

Solution: This says that for all n in the integers, there exists a set X contained in the natural numbers such that the cardinality of X is n. This would be true if it said "For all n in the natural numbers" but as it stands is false, since there are no sets of negative size. (for example let n=-2: is there a subset of the natural numbers that has -2 elements?)