Homework 3 Answers
David Rubinstein - Math 100 - Fall 2019

2.9 - 6) Translate the following sentence into symbolic logic. For every positive number ϵ there is a positive number M such that $|f(x) - b| < \epsilon$ whenever x > M

Solution: This says: $\forall \epsilon > 0, \exists M > 0$ such that $x > M \implies |f(x) - b| < \epsilon$

(2.9- 10) If $sin(x) < 0$ then it is not the case that $0 \leq x \leq \pi$

Solution: $sin(x) < 0 \implies x \notin [0, \pi]$

(2.10- 4) Negate the following sentence: $\forall \epsilon > 0, \exists \delta > 0$ such that $|x - a| < \delta \implies |f(x) - f(a)| < \epsilon$

Solution: I sort of already helped by putting the question in logic symbols. This makes it easier to negate. We get that the negation is: $\exists \epsilon > 0$, such that $\forall \delta > 0, |x - a| < \delta \wedge |f(x) - f(a)| \geq \epsilon$
Note: this original statement is actually the rigorous way to state that the function f is continuous. It is the infamous "epsilon- delta" definition

(2.10- 8) Negate: If x is a rational number and $x \neq 0$ then $tan(x)$ is not a rational number.

Solution: We get that this negation is: For x=0, then tan(x) is rational.

(3.2- 4) In ordering a coffee you have a choice of regular or decaf; small, medium or large; here or to go. How many different ways are there to order a coffee?

Solution: We will use the multiplicative property on this problem. Note that these different choices of coffee can all be formed into a list of size 3 (size, decaf/not, here/to go). Then we have 3 choices for size, 2 for decaf/not and 2 for here/to go; so the multiplicative property tells us that we have $3 \times 2 \times 2 = 12$ different ways of ordering coffee.

(3.2- 10) A dice is tossed four times. How many possible different outcomes are there?

Solution: These outcomes can be made into a length of size 4, and we can again use the multiplicative property to get the total outcomes. In the first toss there are 6 choices, similarly in the second toss, and third toss and fourth toss. Thus we get $6^4 = 1296$ total possible outcomes. (Note if this asked for number of hands you could get all in the same suit it would be $\binom{13}{5} \times 4 = 1287 \times 4 = 5148$, but in our case they said lineup so order matters.)

(3.3- 2) Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same suit?

Solution: We will use the additive principle and the multiplicative principle for this one. Since there are 4 different suits we will find how many lineups there are for each of the suits and then add up the four numbers. For a given suite, we have that we have $13 \times 12 \times 11 \times 10 \times 9 = 154,440$ number of possible lineups. Then since there are 4 suites we get that the total number of lineups in which all 5 cards are of the same suit is $4 \times 154,440 = 617,760$

(3.3- 10) Consider lists of length 6 made from P,R,O,F,S, where repition is allowed. How many such lists can be made if the list must end in an S and the symbol O is used more than once?

Solution: First we find how many lists there are that end in an S, without worrying about the O condition. Then we will subtract off the amount of lists that end in an S but have either 0 or 1 O's in the list. This is a little tricky so let's do this step by step:

First we find there are $5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3125$ number of lists that end in S.

Now we find the amount of lists that have no O's in them: We therefore have a list of length 6 where the last element is an S and we have 4 options for the other entries. Thus we get $4^5 = 1024$ of these lists.

Now we have to find how many lists have just one O in them. This one is more tricky because we have to be careful about where the O is placed. We really have 5 different sets to consider (where the O is in one of the 5 entries in the list) so we can find the amount of lists where the O is in a given place and then multiply it by 5 using the addition principle. For simplicity, assume the O is in the first place in the list. Then we have 4 elements in the list between O and S and we can choose from 4 items for each. Thus we get a total of $4^4 = 256$ lists of this form. Multiplying by 5 we get 1280 lists with one O.

Thus are answer is 3125- 1024- 1280= 821 such lists.

(3.4- 8) Compute how many 7 digit numbers can be made from the numbers 1,2,3,4,5,6,7 if there is no repetition and the odd numbers appear in unbroken sequence.

Solution: We need to only specify where the first odd number is on the list, since the remaining 3 odd numbers will directly follow them. Thus we have the following options; 1st digit (viewed from left) is odd (ex: 1357246); 2nd digit is the first odd, 3rd digit is the first odd; fourth digit is the first odd. Now each of these options will give the same amount of lists (do you see why?), so in order to find the total number, we just find it for one scenario and multiply by 4.

For simplicity, we find how many 7 digit numbers there are where the first digit is odd. We find that we have $4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 = 144$ number of lists. Thus our total is $4 \times 144 = 576$ number of 7 digit numbers with the given requirements.

(3.4- 14) Five of ten books are arranged on a shelf. In how many ways can this be done?

Solution: This is asking us to find how many lists of length 5 there can be without repetition out of a choice of 10 items. In other words, find P(10,5)= $\frac{10!}{5!} = 10\times9\times8\times7\times6 =$ 30.240