Homework 4 Answers
David Rubinstein - Math 100 - Fall 2019

3.5-10) A department consists of 5 men and 7 women. From this department you select a committee with 3 men and 2 women. In how many ways can you do this?

Solution: We get that there are $\binom{5}{3} \times \binom{7}{2} = 210$ ways of doing this. (Come on though, why choose 3 men out of 5 and only 2 out of 7, what the heck guys)

(3.5- 16) How many 6 element subsets of $A = \{0,1,2,3,4,5,6,7,8,9\}$ have exactly 3 even numbers? How many do not have three even elements?

Solution: First we find how many will have 3 even numbers, then by the subtraction principle we can find how many do not.
If there are exactly 3 even numbers in the subset, then are exactly 3 odd numbers as well, so we have the total number of subsets with exactly 3 even numbers is $\binom{5}{3} \times \binom{5}{3} = 100$.
Then since there are $\binom{10}{6} = 210$ subsets of size 6, we get that there are 210-100=110 subsets without exactly 3 even numbers.

(3.5-18) How many 10-digit binary strings have an even number of 1's?

Solution: We will find this by splitting up this problem into smaller pieces, then adding them all together using the addition principle. Let A_0 be the number of 10 digit strings with 0 1's, let A_2 be the number of 10 digit strings with 2 1's, and so on. The goal is to find each A_{2i} and then add them up.
We have that each of these A_{2i} can be formed by choosing the the spots for the 2i number of 1's and then filling the rest of the spots in with 0's. Then we get, for instance that $|A_4| = \binom{10}{4}$. Then the amount of strings with an even number of 1's is $|A_0| + |A_2| + |A_4| + |A_6| + |A_8| + |A_{10}| = \binom{10}{0} + \binom{10}{2} + \binom{10}{4} + \binom{10}{6} + \binom{10}{8} + \binom{10}{10} = 512$.

(3.6-4) Use the binomial theorem to find the coefficient of x^6y^3 of $(3x - 2y)^9$

Solution: The binomial theorem tells us that $(3x - 2y)^9 = \sum_{k=0}^{9} \binom{9}{k}(3x)^{9-k}(-2y)^k$. In our case we want to find the coefficient when k=3. We get that the coefficient is $\binom{9}{3}(3)^6(-2)^3 = -489,888$

(3.6- 12) Show that $\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$

Solution: We first compute the LHS, then the RHS and exhibit that they are equal.
LHS: we get $\binom{n}{k}\binom{k}{m} = \dfrac{n!}{k!(n-k)!}\dfrac{k!}{m!(k-m)!} = \dfrac{n!}{m!(n-k)!(k-m)!}$
RHS: We get $\binom{n}{m}\binom{n-m}{k-m} = \dfrac{n!}{m!(n-m)!}\dfrac{(n-m)!}{(k-m)!(n-m-k+m)!} = \dfrac{n!}{m!(n-k)!(k-m)!}$
Comparing both sides we see they are equal.

(3.7-4) These problems involve making lists from the letters T,H,E,O,R,Y with repition allowed
a)How many 4-letter lists are there that don't begin in T or don't end in Y?

Solution: We first find how many lists there are that don't begin in T, then find how many there are that don't end in Y, and then subtract off the lists that satisfy both (so we don't double count)
We get that there are $5 \times 6^3 = 1080$ lists that don't begin with a T.
Similarly we get $6^3 \times 5 = 1080$ lists that don't end in a Y.
Now we get there are $5 \times 6 \times 6 \times 5 = 900$ lists that don't begin with a T and don't end with a Y.
Hence, we get 1080+1080-900=1260 lists.

b)How many 4-letter lists are there in which the sequence of letters T, H, E appears consecutively (in that order)?

Solution: Let X_1 = the set of lists of the form THE* for some letter *, X_2= the set of lists of the form *THE for some letter *. Then the total number of lists we want is $|X_1| + |X_2| = 6 + 6 = 12$ (since there can be no intersection between X_1, X_2)

c)How many 6-letter lists are there in which the sequence of letters T, H, E appears consecutively (in that order)?

Solution: Again let X_1 =set of lists of form THE***, X_2 =set of lists of form *THE**, X_3 =set of lists of form **THE* and X_4 =set of lists of form ***THE. Now, note that $X_i \cap X_j = \varnothing$ except for when i=1, j=4. Thus we find the total number of lists is $|X_1| + |X_2| + |X_3| + |X_4| - |X_1 \cap X_4| = 6^3 \times 4 - 1 = 863$

(3.7-8) Consider 4-card hands dealt off of a standard 52-card deck. How many hands are there for which all 4 cards are of different suits or all 4 cards are red?

Solution: We find that the number of 4-card hands in which all 4 cards are red is $\binom{26}{4} = 14,950$. Also we get that the number of 4-card hands in which all 4 cards are a different suite is $13^4 = 28,561$. Now it is impossible for there to be any overlap between these two sets, since there are only 2 suites for a red card, so the total number of hands is just 14,950+ 28,561= 43,511

(3.7-12) How many 5-digit numbers are there in which three of the digits are 7, or two of the digits are 2

Solution: Let X_1= the set of 5 digit numbers in which 3 of them are 7 and let X_2= the set of 5 digit numbers in which 2 of them are 2.
To find $|X_1|$ we consider 2 cases: in which the first digit is a 7, and in which the first digit is not a 7
a) If the first digit is a 7, then we have 4 slots to fill in where 2 of them must be a 7. Thus we get $\binom{4}{2} \times 9^2 = 486$ such lists
b) If the first digit is not a 7 then we have 8 options to choose from in the first digit (since the first digit can't be a 0 or 7) and then $\binom{4}{3} * 9$ choices for the last 4 digits, giving us 288 choices.
Hence there are 486+288=774 such lists.
Now we need to find $|X_2|$ and we proceed in a similar way. If the first digit is a 2 then we have $1 \times 4 \times 9^3 = 2,916$ options. If the first digit is not a 2 then we have 8 options for the first slot, $\binom{4}{2} \times 9^2$ for the other 4 slots, giving us 3,888 options in this case. Hence there are 3,888+2,916=6,804 lists in X_2.
Now for $x \in X_1 \bigcap X_2$ we have that x must have 2 twos and 3 fives, and there are $\binom{5}{3} = 10$ ways of that happening.
Hence the total number of 5-digit numbers in which 3 are 5 or 2 are 2 is 774+6,804-10=7568

(3.8-6) A bag contains 20 identical red balls, 20 identical blue balls, 20 identical green balls, one white ball, and one black ball. You reach in and grab 20 balls. How many different outcomes are possible?

Solution: Let $X = \{\text{red, blue, green, white, black}\}$. Then we want to make a list of 20 balls, and can choose between 5 colors to do so. We split this up into multiple cases again.
Case 1: We choose both white ball and black ball. Then we have 18 more choices to

make out of 3 colors so we get $\binom{20}{18} = 190$
Case2: We choose the white marble but not the black marble. Then we have 19 choices out of 3 colors still so we get $\binom{21}{19} = 210$.
Case3: We choose the black marble but not the white marble. This is symmetric to case 2 so we again get 210 choices.
Case4: We don't choose the white or the black marble. Then we get 20 choices between 3 colors so its $\binom{22}{20} = 231$
Adding together all these cases gives us 841.

(3.8-12) How many integer solutions does the equation $w + x + y + z = 120$ if $w \geq 7, x \geq 0, y \geq 5, z \geq 4$

Solution: We have that (w-7)+(x-0)+(y-5)+(z-4)=84. Then we can partition the integers w,x,y,z with a list separated by 3 lines and containing 84 asterisks. Hence we get the total number of integer solutions is $\binom{87}{84} = 105,995$

(3.8-14) How many permutations are there of the letters in the word "PEPPERMINT"?

Solution: We have that this word consists of a multiset of length 10 with multiplicities 3, 2,1,1,1,1,1. Hence we get there are $\frac{10!}{3!2!} = 302,400$