Homework 8 Answers
David Rubinstein - Math 100 - Fall 2019

11.1-2) Let $A = \{1, 2, 3, 4, 5, 6\}$. Write the relation that represents division.

Solution: We have $R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(1,3),(1,4),(1,5),(1,6),(2,4),(2,6),(3,6)\}$

11.1-4) They give you a diagram, and ask to identify A and R

Solution: We have $A = \{0, 1, 2, 3, 4, 5\}$.
We also have $R = \{(0,0),(1,1),(2,2),(3,3),(4,4),(5,5),(0,4),(4,0),(3,1),(1,3),(1,5),(5,1),(2,4),(4,2)\}$

11.2-4) They give you a set A and relation R and ask if it is reflexive, transitive and symmetric.

Solution: It is all 3 lol. This is just definition hunting, so if you know the definition of reflexive, transitive, and symmetric, this is an easy verification.

11.2-14) Suppose R is a symmetric and transitive relation on a set A, and that there exists an $a \in A$ such that aRx for all x in A. Show that R is also reflexive.

Solution: Let $x \in A$. We have that aRx by assumption. Now R is symmetric so this implies we also have xRa. Finally since R is transitive we conclude that xRx, so R is reflexive.

(11.3-2) Let $A = \{a, b, c, d, e\}$. Suppose R is an Equivalence relation on A and R has two equivalence classes. Also suppose aRd, bRc, and eRd. Write out R as a set.

Solution: We have R is reflexive, transitive, and symmetric. Thus we can write R as $R = \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,d),(d,a),(b,c),(c,b),(e,d),(d,e),(a,e),(e,a)\}$

(11.3-6) There are 5 Equivalence relations on the set $A = \{a, b, c\}$

Solution: We get that the ER's are
$R_1 = \{(a,a),(b,b),(c,c)\}$
$R_2 = \{(a,a),(b,b),(c,c),(a,b),(b,a)\}$
$R_3 = \{(a,a),(b,b),(c,c),(a,c),(c,a)\}$
$R_4 = \{(a,a),(b,b),(c,c),(b,c),(c,b)\}$
$R_5 = \{(a,a),(b,b),(c,c),(a,b),(b,a),(a,c),(c,a),(b,c),(c,b)\}$

(11.3-10) Suppose R and S are equivalence relations on a set A. Show that $R \cap S$ is also an equivalence relation on A.

Solution: First assume $R \cap S = \phi$. Then it is vacuously an equivalence relation.
Now assume $R \cap S$ is nonempty; we show it is reflexive, transitive, and symmetric.
Indeed let $x \in A$. Then $xRx \wedge xSx$ so $x(R \cap S)x$.
Now assume $(x,y) \in (R \cap S)$. Then since R and S are both ERs we have $(y,x) \in R$ and $(y,x) \in S$ so $(y,x) \in (R \cap S)$.
Finally, we can show $R \cap S$ is transitive in exactly the same way, as we just showed it is reflexive and symmetric, using the fact that both R and S are ERs.

(11.4-2) List all the partitions on the set $A = \{a,b,c\}$.

Solution: We get that there are 5 partitions,

1. $\{\{a\},\{b\},\{c\}\}$
2. $\{\{a,b\},\{c\}\}$
3. $\{\{a\},\{b,c\}\}$
4. $\{a,c\},\{c\}\}$
5. $\{a,b,c\}$

Note these partitions correspond to the equivalence classes of the equivalence relations we found on problem (11.3-6)

(11.5-4) Write the addition and multiplication tables for $\mathbb{Z}_6$

Solution: We first note that $\mathbb{Z}_6 = \{\overline{0},\overline{1},\overline{2},\overline{3},\overline{4},\overline{5}\}$. We first compute the addition table.

+	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{0}$	$\overline{1}$
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

x	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

11.5-6) Suppose $\overline{a}\overline{b} = \overline{0}$ in $\mathbb{Z}_6$. Is it necessarily true that $\overline{a} = \overline{0}$ or $\overline{b} = \overline{0}$? What if $\overline{a}, \overline{b} \in \mathbb{Z}_7$?

Solution: If $\mathbb{Z}_6$ is is not true that one of $\overline{a}, \overline{b}$ must be 0, since, for example $\overline{23} = \overline{0}$. Howevever, supposose $\overline{ab} = \overline{0} \in \mathbb{Z}_7$. Then we know $7|ab$ and since 7 is prime we have that $7|a$ or $7|b$, ie $\overline{a} = \overline{0}$ or $\overline{b} = \overline{0}$.
This relates to my question3 on section 9 problems I sent you all, about zero divisors. We see that $\mathbb{Z}_p$ behaves as we have intuition for, but $\mathbb{Z}_n$ has some funky behavior.