Midterm Review Guide

August 31, 2021

Question 1- Set theory

Let A={chicken wing, {bagels}, {Bernie Sanders}, $\mathbb{C}$, {$\mathbb{N}$}}
a) Find how many subsets A has
b) Is $\mathbb{N} \in A$? Is {bagels} $\subseteq A$?
c) Now let B= {Spanish, Hebrew, {14}, R-modules} Find $A \times B$
d) Find $|\mathcal{P}(\mathcal{P}(A) \times B)|$
e) Graph $\{x \in [-2\pi, 2\pi] : cos(x) = 0\} \times [-5, 5]$

Question 2- Logic

i)Determine if the following sentences are $P \implies Q, Q \implies P, P \iff Q$
Make sure to explain what your P and Q are
a) An infinite series converges only if the limit of its terms goes to 0.
b) In determining whether a list of n-vectors in a given n-dimensional vector space form a basis for the vector space, it is necessary and sufficient to check that they are Linearly independent.
c) A ring is Noetherian if it is Artenian.

ii)Write out the following sentences in English, and decide whether it is true or false:
a)$\forall n \in \mathbb{N}, \exists, X \in \mathcal{P}(\mathbb{N}), |X| < n$
b) $\exists n \in \mathbb{N}, \forall X \in \mathcal{P}(\mathbb{N}), |X| < n$

Question 3- Counting

i)Consider lists of length 4 from A,B,C,D,E,F,G

a) How many lists are there if repetition is allowed?
b) How many lists are there if repetition not allowed and must have a G?
c) How many lists are there if repetition allowed and must have G?

ii) A set X has exactly 84 subsets of size 3.
a) What is the cardinality of X?
b) How many subsets of size 5 does X have?

iii)
a)Find the coefficient in front of x^8 for $(-2x-3)^{12}$
b)Show that $4^n = \sum_{k=0}^{n} \binom{n}{k} 3^k$

iv) Consider 4-card hands dealt off of a standard 52-card deck. How many hands are there for which all 4 cards are of the same suit or all 4 cards are red?

v) How many integer solutions are there to the equation u+v+x+y+w=90 if u,v,x,y,w ≥ 0?

vi) There is a Yiddish word tchatchke: Find how many permutations there are of the letters in this word.

Question 4- Proofs
Prove the following:
a) If a is an odd integer then $a^2 + 3a + 5$ is odd
b) If two numbers are of opposite parity then their product is even.
c) If $n \in \mathbb{Z}$ then $n^2 + 3n + 4$ is even.
d) Suppose a,b $\in \mathbb{Z}$. If $a^2(b^2 - 2b)$ is odd, then a and b are both odd.
e) Let a,b $\in \mathbb{Z}, n \in \mathbb{N}$. Show that if $a \equiv b$ (mod n) then $a^3 \equiv b^3$ (mod n).