Introduction to Logic Questions

August 31, 2021

Question 1

Decide whether the following are statements: if they are statements determine whether they are true or false:

  a) The derivative of sin(x) is cos(x)
  b) Tell your mom you love her
  c) Corn cake is a vegetable
  d) Give him water
  e) $\mathbb{Q} \in \mathcal{P}(\mathbb{R})$

Question 2
Without changing their meanings, convert the following sentences into a sentence having the form "If P then Q". Make sure to explain what P and Q are:

  a) For a list of k different vectors to be linearly independent in $\mathbb{R}^n$ we must have $k \leq n$
  b) The geometric series converges only if the ratio $|r| < 1$
  c) David will be happy only if you get this question right!!!!

Question 3
Figure out if these statements are of the form $P \implies Q, Q \implies P$ or $P \iff Q$. Express them as such:

  a) For a number to be prime, it is necessary that no other number divide it
  b) To find a basis in an n dimensional vector space, one must only find n-linearly independent vectors
  c) The equation $x^n + y^n = z^n$ has an integer solution if and only if $n \leq 2$ (note: this is a famous theorem that took about 300 years to prove)

Question 4- Challenge Question

a) Find a way to write $P \iff Q$ only in terms of $\wedge, \vee$, and $\neg$ (where $\neg$ is "not")

b) Prove $\neg(P \vee Q) = \neg P \wedge \neg Q$ and $\neg(P \wedge Q) = \neg P \vee \neg Q$

c) Prove $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ and $P \vee (Q \wedge P) = (P \vee Q) \wedge (P \vee R)$ This shows us we have to be careful about the order we do these when we mix ands, and ors. (One can also show these operations are commutative and associative)