Introduction to Counting (lol)

August 31, 2021

Question 1

Consider a list of 8 letters A,B,C,D,E,F,G,H. Find

a) How many lists of length 4 are there if repetition is allowed
b) How many lists of length 4 are there if no repetition is required
c) How many lists of length 4 are there if no repetition and the first and last letter are vowels.

Question 2

Solve the following:

a) How many integers between 1 and 99 have no repeated digits
b) How many integers between 1 and 1000 are divisible by 5
c) How many integers between 1 and 1000 are not divisible by 5

Question 3

b) The all powerful King David wants to choose a cabinet to help him run his kingdom; He wants to appoint a secretary of snacks, a minister of sports, and a president of national parks. Twenty people apply to these positions, how many different ways can the magnificent King David fill these positions?

c) The new minister of snacks that the magnificent King David recently appointed is deciding on the snacks to supply to the kingdom: They have narrowed the choice down to 15 chocolates and 15 candies. Kind David demands only 10 options be present at the royal court: how many permutations of the snacks are there if the secretary of snacks must switch between chocolate and non chocolate options?

Question 4- Challenge Question

There is a wonderful way in which one can generalize the idea of factorial. It has to do with the Gamma function $\Gamma : [0, \infty) \to \mathbb{R}$ defined as follows: $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. This doesn't seem to have any relationship to the factorial as we have defined it, but a remarkable fact is that if $x \in \mathbb{N}$ then $\Gamma(x) = (x-1)!$. Thus we get a way to generalize the idea of factorial to any real number, not restricted to just the natural numbers. Play around and try to show that $\Gamma(x) = (x-1)!$ for some small integer values of x. Once you do, try and calculate π!