

Finishing up Counting Discussion

August 31, 2021

Question 1

A set X has exactly 56 subsets with 3 elements in it.

- What is the cardinality of X ?
- How many subsets with 4 elements does X have?

Question 2

Solve the following:

- What is the coefficient of x^5 in $(x + 4)^7$
- What is the coefficient of x^6 in $(3x - 2)^8$
- Show $9^n = \sum_{k=0}^n (-1)^k 10^{n-k}$

Question 3

- How many 4-digit positive integers are there that are even or contain no 0's?
- A bag contains 20 identical red balls, 20 identical blue balls, 20 identical green balls, and 20 identical white balls. You reach in and grab 15 balls. How many different outcomes are possible??

Question 4- Challenge Question

There is a method of proving that two formulas are equal, called the "Combinatorial proof". The gist is that you show both formulas are really counting the same thing. For example, we showed in class that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

by showing that both are counting the number of k -element subsets of an $n+1$ element set. Using the methods of combinatorial proof, show that $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$ for any positive integer n .