

Pigeonhole Theorem/ Intro to Proofs

August 31, 2021

Question 1

- a) A martian has an infinite amount of red, blue, yellow and black socks. How many socks must they pull out of the drawer to guarantee they have a pair?
- b) Assume California has a population of 40 million people. Show that there are at least 7 people who were born on the same day of the year.
- c) Use Combinatorial proof to show that $\binom{n}{k} = \binom{n}{n-k}$

Question 2

- a) Suppose $a, b, c \in \mathbb{Z}$. Show that if $a^2|b$ and $b^3|c$ then $a^6|c$.
- b) Suppose x is odd. Show that $x^2 + 3x + 9$ is even.
- c) Suppose $a|b$. Show that $a|(b^3 - 3b^2 + 6b)$.
- d) More generally, show that if $a|b$ then $a|(a_n b^n + a_{n-1} b^{n-1} + \dots + a_1 b)$ for any $a_1, \dots, a_n \in \mathbb{Z}$ and $n \in \mathbb{N}$

Question 3- Challenge Question

We will prove this later but we really have most of the tools already to prove this: Show that there are an infinite amount of prime numbers. (Hint: This proof uses the method of contradiction. Namely; assume for the sake of contradiction that there are only finitely many prime numbers. Show that you can then construct a new one that is not in your finite list, contradicting that the list was finite.)