Eigenvalues, Eigenvectors and Diagnolization

August 20, 2021

Question 1

Find the eigenvalues and eigenvectors of the following matrices.

a) $\begin{pmatrix} 2 & -8 \\ -2 & -4 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -12 & 0 & 0 & 0 \\ 0 & 0 & -12 & 0 & 0 \\ 0 & 0 & 0 & 92 & 0 \\ 0 & 0 & 0 & 0 & -120 \end{pmatrix}$

Question 2
State whether the following are true or false. If false, explain why or give a counter-example.

a) Suppose $T : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ is a linear transformation with eigenvalues $\lambda_1 = 1, \lambda_2 = -2, \lambda_3 = -12$. Then T is an isomorphism.
b) A given eigenvector has only 1 eigenvalue associated to it.
c) Suppose A is an $n \times n$ matrix, and λ is an eigenvalue for A. Then the columns of $(A - \lambda I_n)$ are linearly independent.
d) A given eigenvalue has only 1 eigenvector associated to it.

Question 3

Let $\mathcal{B} = (1, x, x^2)$ be the standard basis for $\mathbb{R}_2[x]$, and suppose

$$T : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$$

is a linear transformation whose matrix with respect to $\mathcal{B}$ is

$$A_{T,\mathcal{B}} = \begin{pmatrix} 5 & 2 & -4 \\ 6 & 3 & -5 \\ 10 & 4 & -8 \end{pmatrix}$$

We showed in class that this matrix has the following eigenvectors with associated eigenvalues;

$$v_1 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} \text{ with } \lambda_1 = -1$$

$$v_2 = \begin{pmatrix} \frac{1}{2} \\ 1 \\ 1 \end{pmatrix} \text{ with } \lambda_2 = 1$$

$$v_3 = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 1 \end{pmatrix} \text{ with } \lambda_3 = 0$$

a) Show that $\mathcal{C} = (v_1, v_2, v_3)$ is a basis for $\mathbb{R}^3$.
b) Let $\mathcal{S} = (e_1, e_2, e_3)$ be the standard basis for $\mathbb{R}^3$. Find

$$\mathcal{P}_{\mathcal{S} \to \mathcal{C}} \quad (1)$$

$$\mathcal{P}_{\mathcal{C} \to \mathcal{S}} \quad (2)$$

.

c) Find the matrix multiplication

$$D = (\mathcal{P}_{\mathcal{S} \to \mathcal{C}})(A_{T,\mathcal{B}})(\mathcal{P}_{\mathcal{C} \to \mathcal{S}})$$

d) What is the relationship of this matrix D with respect to the original transformation T?