

# Spanning, Linear Independence, and Bases

July 28, 2021

Question 1

- a) Is the vector  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$  a linear combination of the vectors  $\left( \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right)$
- b) Is the vector  $\begin{pmatrix} 2 \\ 8 \\ 11 \end{pmatrix}$  a linear combination of the vectors  $\left( \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \\ 6 \end{pmatrix}, \begin{pmatrix} -1 \\ -12 \\ -1 \end{pmatrix} \right)$
- c) Is the vector  $\begin{pmatrix} 1 \\ -1 \\ -8 \end{pmatrix}$  a linear combination of the vectors  $\left( \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -4 \end{pmatrix} \right)$

Question 2

Answer the Challenge question from HW 1. (You can now answer it quickly, once you rephrase it in terms of spanning vectors)

Question 3

For the following, give an example if one exists, or state it is not possible. If it is not possible, explain why.

- a) A sequence of 3 vectors in  $\mathbb{R}^3$  that are LI.
- b) A sequence of 2 vectors in  $\mathbb{R}^3$  that are spanning vectors.

- c) A sequence of 4 vectors in  $\mathbb{R}^2$  that are spanning vectors.
- d) A sequence of 3 vectors that are a basis for  $\mathbb{R}^3$
- e) A sequence of 3 vectors that are a basis for  $\mathbb{R}^4$

Question 4

State whether the following are true or false. For those that are false, give a counter example.

- a) Every list of two vectors in  $\mathbb{R}^3$  are LI.
- b) Every list of four vectors in  $\mathbb{R}^3$  are spanning.
- c) Every list of four linearly independent vectors in  $\mathbb{R}^4$  are a basis.
- d) Every list of five vectors in  $\mathbb{R}^5$  that span  $\mathbb{R}^5$  are a basis.