

Invertible and Elementary Matrices

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Question 1

For the following, give an example if one exists, or explain why no such example exists.

- a) A 3×3 matrix which has a nonzero vector in its null space.
- b) An invertible 4×4 matrix whose columns do not span \mathbb{R}^4
- c) An invertible 3×3 matrix A , along with two 3×3 matrices B, C such that $AB=AC$ but $B \neq C$
- d) Two nonzero 3×3 matrices A, B such that $AB=0_{3 \times 3}=BA$ (where $0_{3 \times 3}$ is the 3×3 matrix of all 0's)

Question 2

Determine if the following matrices are invertible. If they are invertible find their inverse.

a) $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$.

c) $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 3 \end{pmatrix}$.

Question 3

Consider the matrix $A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$. First, compute the following three matrix multiplications

$$A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (1)$$

$$A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad (2)$$

$$A \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3)$$

State how these three matrices you get after computing the multiplication are related to the original matrix A . Is there a pattern, and can a general result be conjectured from this? (Hint, the matrices you are asked to multiply A by are elementary matrices: what happens when you multiply a matrix by an elementary matrix on the left?)